

Math 418: Problem Set 4.

Due date: In class on Wednesday, February 24.

Webpage: <http://dunfield.info/418>

Office hours: Monday 10-11, Tuesday 3-5.

All problems are from Dummit and Foote, *Abstract Algebra*, 3rd edition.

1. Let K/F be an algebraic extension. Suppose R is a *subring* contained in K which contains F . Prove that R is actually a *subfield* of K .
2. Prove that $\alpha = \cos(2\pi/5)$ is a constructable number. Use this to show that the regular 5-gon is constructable by straightedge and compass.
3. Find the splitting field K of $x^4 - 2$ over \mathbb{Q} . What is $[K : \mathbb{Q}]$?
4. Find the splitting field K of $x^4 + x^2 + 1$ over \mathbb{Q} . What is $[K : \mathbb{Q}]$?
5. Suppose K/F is the splitting field for a polynomial $f(x) \in F[x]$. Let $g(x) \in F[x]$ be irreducible. Show that if g has a root in K then it splits completely in $K[x]$.