1. Let $\mathbf{u} = \langle 1, 0, 2 \rangle$, $\mathbf{v} = \langle -3, 1, 1 \rangle$, and $\mathbf{w} = \langle 2, -1, 1 \rangle$ be vectors in $\mathbb{R}^3$.

(a) Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$. Circle the value of θ below. **(2 points)**

$\theta = 0$	$0 < \theta < \pi/2$	$\theta = \pi/2$	$\pi/2 < \theta < \pi$	$\theta = \pi$

(b) Circle the value of $|2\mathbf{u} - \mathbf{v}|$. **(2 points)**

$\sqrt{10}$	$\sqrt{11}$	$\sqrt{30}$	$\sqrt{34}$	$\sqrt{35}$

(c) Mark the answer that best describes the meaning of the expression $|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$. **(1 point)**

- [ ] It is the sum of the areas of the two parallelograms determined by the two pairs $\{\mathbf{u}, \mathbf{v}\}$ and $\{\mathbf{u}, \mathbf{w}\}$.
- [ ] It is the area of the parallelogram with vertices determined by 0, $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$.
- [x] It is the volume of the parallelepiped determined by the vectors $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$.
- [ ] It has no meaning, but it is always defined and sometimes it is zero.
- [ ] It is undefined.

Scratch Space

a) $\vec{u} \cdot \vec{v} = 1 \cdot (-3) + 0 \cdot 1 + 2 \cdot 1 = -1$

$|\vec{u}| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}$ $\quad |\vec{v}| = \sqrt{9+1+1} = \sqrt{11}$

So $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-1}{\sqrt{55}}$ is in $(-1, 0)$ and hence $\pi/2 < \theta < \pi$.

b) $2u - v = \langle 2, 0, 4 \rangle - \langle -3, 1, 1 \rangle = \langle 5, -1, 3 \rangle$

So $|2u - v| = \sqrt{5^2 + (-1)^2 + 3^2} = \sqrt{35}$

2. Find a parametric equation for the line passing through the points $A(2,0,3)$ and $B(3,1,6)$.

Take $\vec{v} = \overrightarrow{AB} = (3,1,6) - (2,0,3) = (1,1,3)$

Then can param by $\vec{r}(t) = (2,0,3) + t\vec{v}$

$= (2+t, t, 3+3t)$

$\boxed{(x(t), y(t), z(t)) = (\ 2+t \ ,\ t \ ,\ 3+3t\ )}$

3. Find an equation of the plane that contains the line $\mathbf{r}(t) = (1+t,\ 1,\ 2t)$ and the point $P(2,\ 4,\ 0)$.

L parameterized by

The point $Q = \vec{r}(0) = (1,1,0)$ is on L and $\vec{a} = \langle 1,0,2 \rangle$ points along it. Thus $\vec{a}$ and $\vec{b} = \overrightarrow{QP} = \langle 1,3,0 \rangle$ are in the plane and we can use the following normal

$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 1 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ 3 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix}\vec{k}$$

$$= \langle -6, 2, 3 \rangle$$

Thus using Q as our pt on the plane we get

$$-6(x-1) + 2(y-1) + 3(z-0) = 0$$

as the eqn, which is also

Equation: $\boxed{-6}\,x + \boxed{2}\,y + \boxed{3}\,z = \boxed{-4}$

4. Let **a** and **b** be two vectors in $\mathbb{R}^3$ such that $|\text{proj}_{\mathbf{a}}\mathbf{b}| = 2$. **(1 point each)**

(a) Determine the value of $|\text{proj}_{3\mathbf{a}}\mathbf{b}|$. Circle your answer:

$\frac{2}{3}$	2	3	5	6

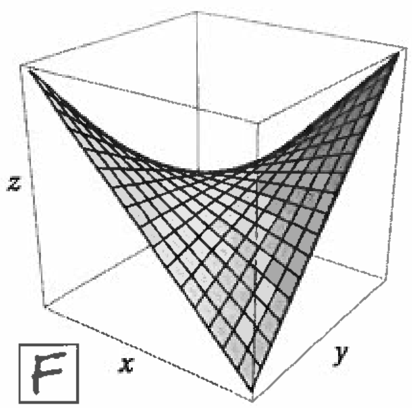
(b) Determine the value of $|\text{proj}_{\mathbf{a}}5\mathbf{b}|$. Circle your answer:

$\frac{2}{5}$	2	5	7	10

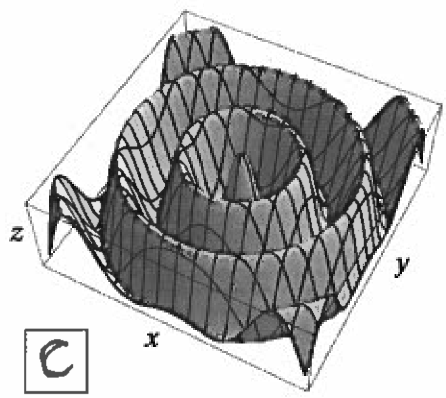
5. Find the value of m such that the vector $\mathbf{u} = \langle -9, m, 6\rangle$ is perpendicular to the plane $3x + y - 2z = 15$. Circle your answer. **(2 points)**

$m = -3$	$m = -1$	$m = 0$	$m = 1$	$m = 39$

6. For each graph below, find the corresponding function from the options at right and write the corresponding letter in the box next to the graph. **(2 points each)**



F



C

(A) $\sin(x+y)$

(B) $x^2 - y^2$

(C) $\cos\left(\sqrt{x^2+y^2}\right)$

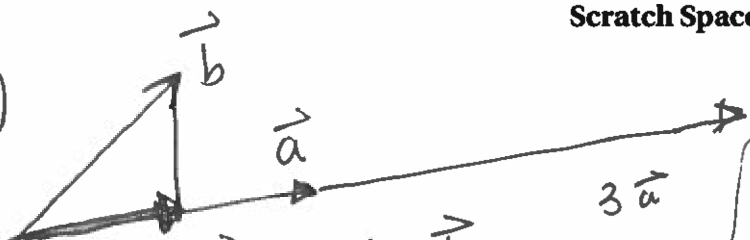
(D) $\cos(x)\cos(y)$

(E) $(x-y)^2$

(F) xy

Scratch Space

4) a)



$\text{proj}_{\vec{a}}\vec{b} = \text{proj}_{3\vec{a}}\vec{b}$

b) $\text{proj}_{\vec{a}}5\vec{b} = 5\,\text{proj}_{\vec{a}}\vec{b}$

5) Want $\vec{u}$ to be a scalar mult of the normal $\vec{n} = \langle 3, 1, -2\rangle$ of the plane. If $\vec{u} = \langle -9, m, 6\rangle = s\vec{n} = \langle 3s, s, -2s\rangle$ we get $-9 = 3s$ and $m = s \Rightarrow s = m = -3$.

7. Let $f(x,y) = x^3y + 2xy^2 + y$.

(a) Find the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(0,1,1)$.

$f_x = 3x^2y + 2y^2$ at $(0,1)$ is 2

$f_y = x^3 + 4xy + 1$ at $(0,1)$ is 1

Eqn for plane is

$$(z - f(0,1)) = f_x(0,1)(x-0) + f_y(0,1)(y-1)$$

$$\iff z - 1 = 2x + y - 1$$

$$\iff 2x + y - z = 0$$

Equation: $\boxed{2}\,x + \boxed{1}\,y + \boxed{-1}\,z = \boxed{0}$

(b) Use linear approximation to estimate the value of $f(0.2, 0.9)$.

$$f(0.2, 0.9) \approx f(0,1) + f_x(0,1)\cdot 0.2 + f_y(0,1)\cdot(-0.1)$$

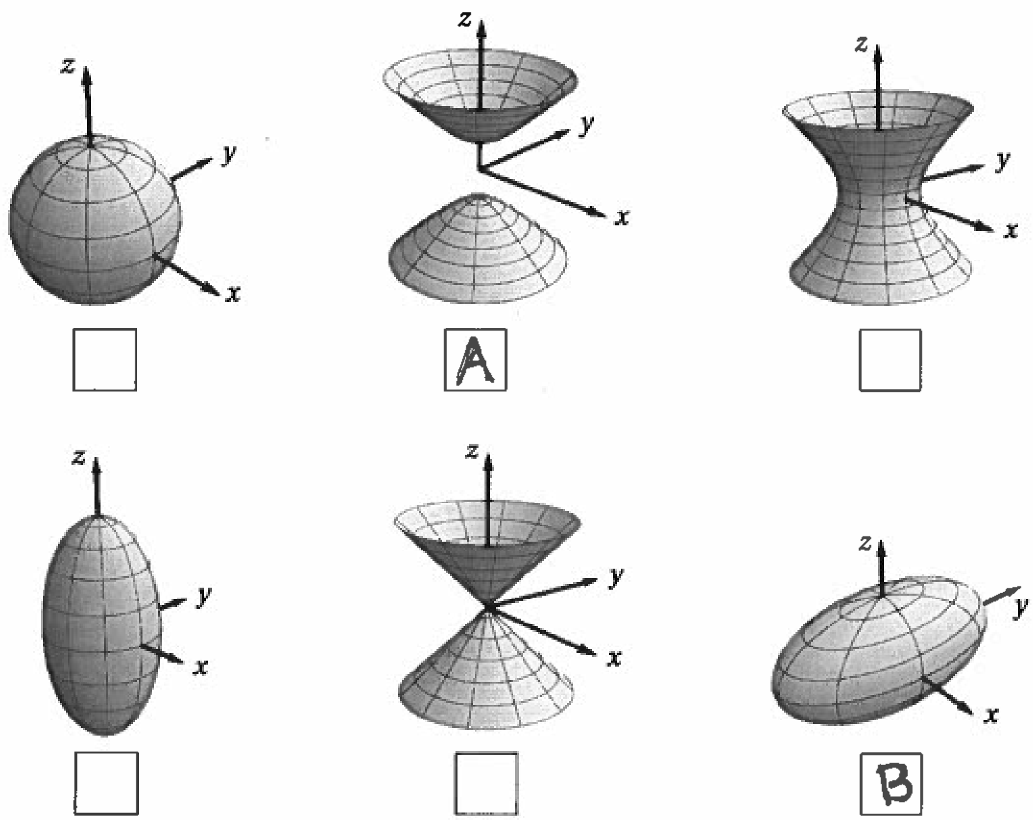
$$= 1 + 2\cdot 0.2 + 1\cdot(-0.1)$$

$$= 1.3$$

$f(0.2, 0.9) \approx$ 1.3

8. For each equation below, write the corresponding letter in the box next to the picture of the surface it describes. **(2 points each)**

(A) $x^2+y^2-z^2+1=0$ (B) $4x^2+y^2+4z^2-1=0$

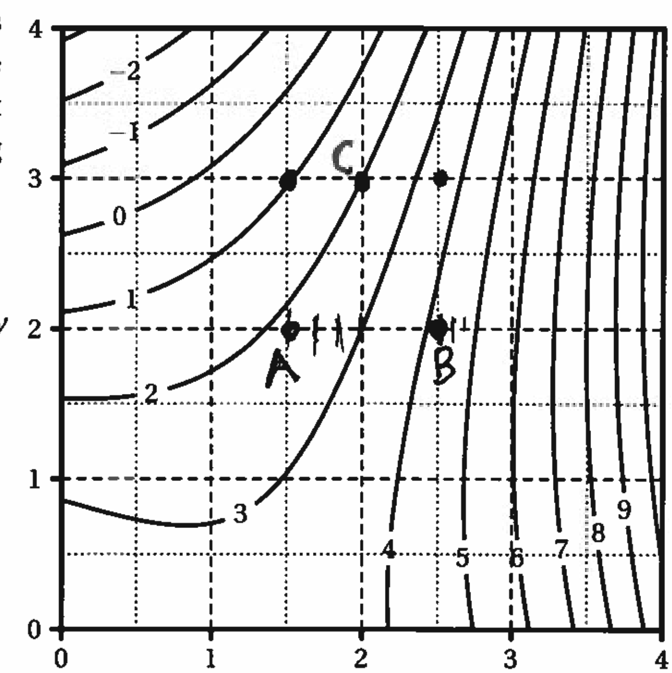


A) $x^2+y^2+1=z^2$

$z=\pm\sqrt{1+x^2+y^2}$

B) This is an ellipsoid containing $(1/2,0,0)$ $(0,1,0)$ and $(0,0,1/2)$

9. The contour map of a differentiable function $f(x,y)$ is shown at right, where each level curve is labeled by the corresponding value of f. For each part, circle the best possible answer. **2 points** for part (a) and the remaining parts are **1 point each.**



(a) $\dfrac{\partial f}{\partial x}(2,2)$ is

−4	−2	−1	0	1	2	4

(b) $\dfrac{\partial f}{\partial y}(2,2)$ is negative zero positive

(c) $\dfrac{\partial^2 f}{\partial x^2}(2,2)$ is negative zero positive

(d) $\dfrac{\partial^2 f}{\partial y \partial x}(2,2)$ is negative zero positive

since $\dfrac{\partial f}{\partial x}(C) \approx 2.5$

$\dfrac{f(B)-f(A)}{\Delta x} = \dfrac{4.25-2.25}{1} = 2$

10. Let $f(x,y) = \dfrac{xy^2}{3x^2+y^4}$. Determine the limits in the problems below. Be sure to *explain your reasoning.* If a limit does not exist, write "DNE" in the box provided.

(a) Determine $\lim_{y\to 0} f(y^2, y)$.

$$\boxed{\lim_{y\to 0} f(y^2,y) = 1/4}$$

$$\lim_{y\to 0} f(y^2, y) = \lim_{y\to 0} \frac{y^2\cdot y^2}{3y^4+y^4} =$$

$$\lim_{y\to 0} 1/4 = 1/4$$

(b) Determine $\lim_{(x,y)\to(0,0)} f(x,y)$. If we approach $(0,0)$ along the line $y=x$ we get $\lim_{x\to 0} \frac{x\cdot x^2}{3x^2+x^4}$ $= \lim_{x\to 0} \frac{x}{3+x^2} = \frac{0}{3} = 0.$

However, by (a) if we approach along the parabola $x=y^2$ we get a limit of $1/4$.

Hence:

$$\boxed{\lim_{(x,y)\to(0,0)} f(x,y) = \text{DNE}}$$

(c) Determine $\lim_{(x,y)\to(1,0)} f(x,y)$. Both the numerator and denominator are continuous, and the denominator is nonzero at $(1,0)$. Hence can evaluate just by plugging in: $\frac{1\cdot 0^2}{3\cdot 1^2+0^2} = 0$

$$\boxed{\lim_{(x,y)\to(1,0)} f(x,y) = 0}$$

11. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function of two variables. Let $x(u,v) = u^2 v$ and $y(u,v) = u\cos(v)$. Consider the function $g(u,v) = f\big(x(u,v), y(u,v)\big)$. Use the table of values for f and g below to compute $g_u(1,0)$. **(4 points)**

	g	f	f_x	f_y	f_{xx}	f_{xy}
(1,0)	5	1	2	-1	0	5
(0,1)	0	5	3	-2	10	-11

Chain Rule:

$$\frac{\partial g}{\partial u}(1,0) = \frac{\partial f}{\partial x}\big(\underbrace{x(1,0), y(1,0)}_{=(0,1)}\big)\frac{\partial x}{\partial u}(1,0) +$$

$$\frac{\partial f}{\partial y}\big(x(1,0), y(1,0)\big)\frac{\partial y}{\partial u}(1,0)$$

$$= 3\cdot 0 + (-2)\cdot 1 = -2$$

$\frac{\partial x}{\partial u} = 2uv$ is 0 at $(u,v) = (1,0)$

$\frac{\partial y}{\partial u} = \cos v$ is 1 at $(u,v) = (1,0)$

$g_u(1,0) = \boxed{-2}$

Scratch Space