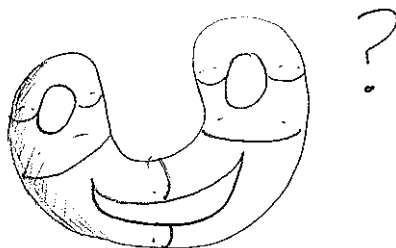
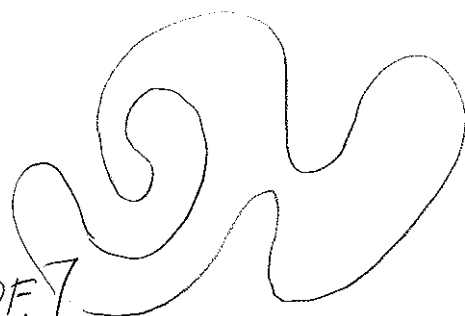
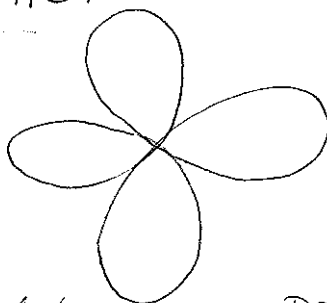
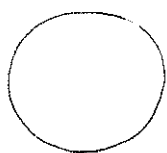
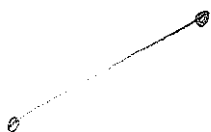


Q: How to parameterize the surface of a 787?  
or a tea pot? or



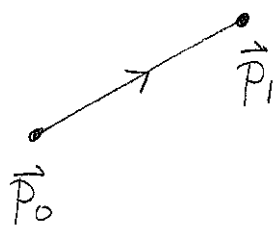
Start with curves in  $\mathbb{R}^2$ :



[Show smooth curves of letters in a PDF.]

Bézier Curves: [1960s Renault autos.]

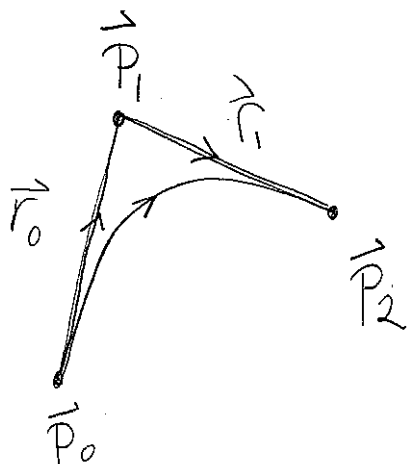
Linear:



$$\vec{r}(t) = (1-t)\vec{p}_0 + t\vec{p}_1$$

"Interpolation."

Quadratic:



Goals: Initially head from  $p_0$  to  $p_1$ , and come in to  $p_2$  as if coming from  $p_1$ .

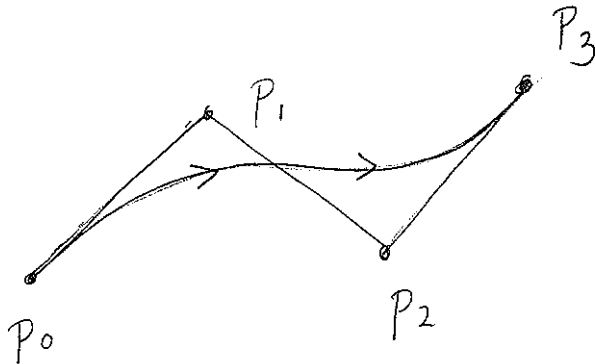
$$\begin{aligned}
 \vec{r}(t) &= (1-t)\vec{r}_0(t) + t\vec{r}_1(t) \\
 &= (1-t)((1-t)\vec{p}_0 + t\vec{p}_1) + t((1-t)\vec{p}_1 + t\vec{p}_2) \\
 &= (1-t)^2\vec{p}_0 + 2t(1-t)\vec{p}_1 + t^2\vec{p}_2
 \end{aligned}$$

Sample Checks:  $\vec{r}(0) = \vec{p}_0$

$$\vec{r}'(t) = -2(1-t)\vec{p}_0 + 2(1-2t)\vec{p}_1 + 2t^2\vec{p}_2$$

$$\vec{r}'(0) = 2(\vec{p}_1 - \vec{p}_0)$$

Cubic:



$$\vec{r}(t) = (1-t)\vec{r}_{P_0, P_1, P_2}(t) + t\vec{r}_{P_1, P_2, P_3}(t)$$

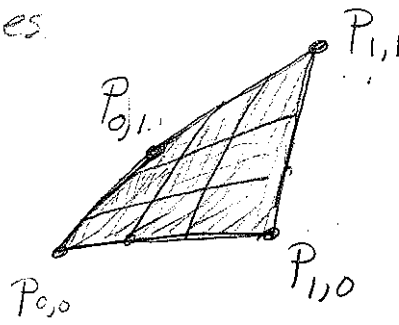
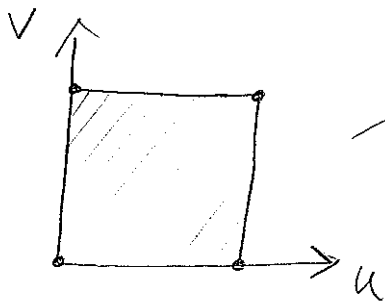
Can do more, but for most things this is enough.

Point: Humans can't see discontinuities in 3<sup>rd</sup> derivatives.

Note: Transform nicely under linear transformations.

# On to 3<sup>d</sup> Bézier Patches

(2)

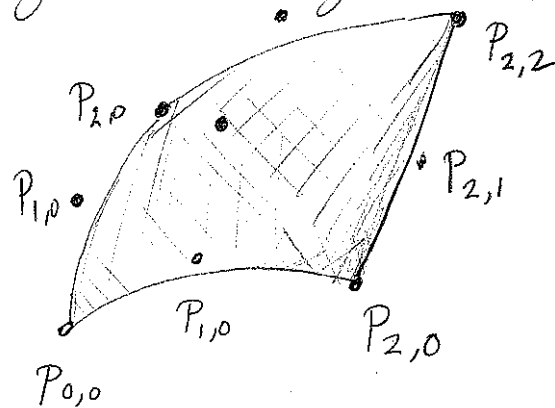


Simplist type:

$$\vec{r}(u,v) = (1-u)(1-v) \vec{P}_{0,0} + u(1-v) \vec{P}_{1,0} + v(1-u) \vec{P}_{0,1} + uv \vec{P}_{1,1}$$

Properties: straight lines go to straight lines.

Biquadratic:



$$\begin{aligned} \vec{r}(u,v) = & (1-u)^2(1-v)^2 P_{0,0} + 2u(1-u)(1-v)^2 P_{1,0} + u^2(1-v)^2 P_{2,0} \\ & + 2(1-u)^2 v(1-v)^2 P_{0,1} + 4u(u-1)v(v-1) P_{1,1} + 2u^2 v(1-v) P_{2,1} \\ & + (1-u)^2 v^2 P_{0,2} + 2u(u-1)v^2 P_{1,2} + u^2 v^2 P_{2,2} \end{aligned}$$

Typical: 16 control points.

Problem: • Matching adjacent patches.

- Issue in computational fluid dynamics.
- Finding intersection of two patches.
- Preserving the topology...

