

Thursday, August 29 \*\* Parametric curves defined using vector arithmetic.

1. Let  $f(x) = x^2 + x - 2$ .

- (a) Graph the equation  $y = f(x)$ . (By hand, then check with a calculator if you want.)
- (b) Find the slope and equation of the tangent line to  $y = f(x)$  when  $x = 2$ . Draw the tangent line on your picture.
- (c) Draw a vector in  $\mathbb{R}^2$  that describes the direction of the line. Find a numeric representation of your vector.

2. Consider the curve given parametrically by

$$\begin{cases} x(t) = t \\ y(t) = t^2 + t - 2 \end{cases} \quad \text{for } 0 \leq t < 4.$$

- (a) Sketch the curve. How does this graph differ from your graph in Problem 1(a)?
- (b) Consider the vectors formed by the pair  $(x(t), y(t))$ . Anchoring the vectors at the origin, sketch on your graph the vectors at time  $t = 0, 1, 2, 3$ .
- (c) Now consider the vectors formed by  $(x'(t), y'(t))$ . Evaluate  $(x'(t), y'(t))$  at time  $t = 2$ , what does the vector represent? Hint: Graph it on the curve at the point  $(x(2), y(2))$ .
- (d) Imagine that the curve is the path of a moving particle. What is the speed of the particle when  $t = 2$ ?

3. (a) Sketch the vector emanating from the origin ending at the point  $(-5, 2)$  in  $\mathbb{R}^2$ .
- (b) On the same graph and using the “head-to-tail” geometric addition method, draw the vector  $(-5, 2) + (3, -1)$ .
- (c) Do the same for  $(-5, 2) + 2(3, -1)$ .
- (d) Do the same for  $(-5, 2) + (-1)(3, -1)$ .
- (e) If we allow the scalar multiplying the vector  $(3, -1)$  to vary, what geometric object is described by the parametric equation  $(-5, 2) + t(3, -1)$  for all  $t$ ?

4. Consider the set of points in  $\mathbb{R}^3$  defined by the parametric equation

$$\mathbf{l}(t) = (-5 + 2t, 2 + 3t, 1 - t) \quad \text{for all } t.$$

- (a) Using the properties of vector arithmetic, factor  $\mathbf{l}(t)$  into the form  $\mathbf{p} + t\mathbf{v}$  where  $\mathbf{p}$  and  $\mathbf{v}$  are vectors in  $\mathbb{R}^3$ .
- (b) Using the factored form (and your technique from Problem 3) sketch this object in  $\mathbb{R}^3$ . Geometrically, what does this parametric equation describe?
- (c) Why is the vector  $\mathbf{v}$  in your factored form referred to as the *direction vector*?

5. Let  $\mathbf{a} = (-\sqrt{3}, 0, -1, 0)$  and  $\mathbf{b} = (1, 1, 0, 1)$  be vectors in  $\mathbb{R}^4$ .

- (a) Find the distance between the points  $(-\sqrt{3}, 0, -1, 0)$  and  $(1, 1, 0, 1)$ .
- (b) Find the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .