**Tuesday, October 1** $**$ *Taylor series, the* $2^{\text{nd}}$ *derivative test, and changing coordinates.*

1. Consider $f(x,y) = 2\cos x - y^2 + e^{xy}$.
   (a) Show that $(0,0)$ is a critical point for $f$.
   (b) Calculate each of $f_{xx}$, $f_{xy}$, $f_{yy}$ at $(0,0)$ and use this to write out the $2^{\text{nd}}$-order Taylor approximation for $f$ at $(0,0)$.
   (c) To make sure the next two problems go smoothly, check your answer to (b) with the instructor.
2. Let $g(x,y)$ be the approximation you obtained for $f(x,y)$ near $(0,0)$ in 1(b). It's not clear from the formula whether $g$, and hence $f$, has a min, max, or a saddle at $(0,0)$. Test along several lines until you are convinced you've determined which type it is. In the next problem, you'll confirm your answer in two ways.
3. Consider alternate coordinates $(u,v)$ on $\mathbb{R}^2$ given by $(x,y) = (u-v, u+v)$.
   (a) Sketch the $u$- and $v$-axes relative to the usual $x$- and $y$-axes, and draw the points whose $(u,v)$-coordinates are: $(-1,2)$, $(1,1)$, $(1,-1)$.
   (b) Express $g$ as a function of $u$ and $v$, and expand and simplify the resulting expression.
   (c) Explain why your answer in 3(b) confirms your answer in 2.
   (d) Sketch a few level sets for $g$. What do the level sets of $f$ look like near $(0,0)$?
   (e) It turns out that there is always a similar change of coordinates so that the Taylor series of a function $f$ which has a critical point at $(0,0)$ looks like $f(u,v) \approx f(0,0) + au^2 + bv^2$. In fact this is why the $2^{\text{nd}}$ derivative test works.

      Double check your answer in 2 by applying the $2^{\text{nd}}$-derivative test directly to $f$.
4. Consider the function $f(x,y) = 3xe^y - x^3 - e^{3y}$.
   (a) Check that $f$ has only one critical point, which is a local maximum.
   (b) Does $f$ have an absolute maxima? Why or why not? Check your answer with the instructor.