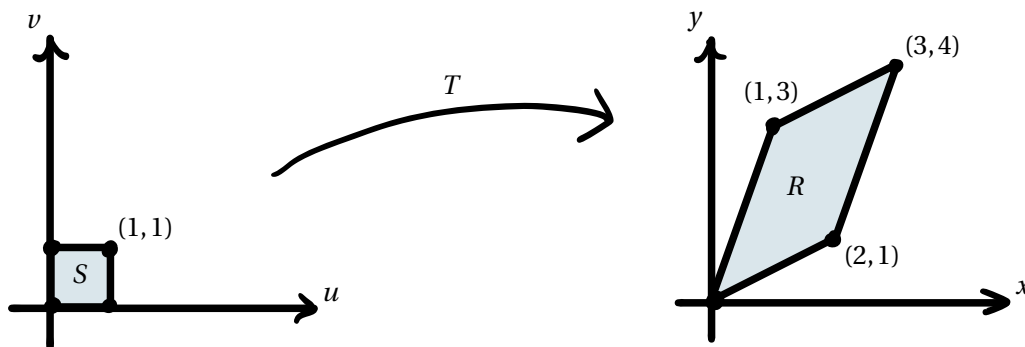


Tuesday, November 5 \*\* Changing coordinates

1. Consider the region  $R$  in  $\mathbb{R}^2$  shown below at right. In this problem, you will do a change of coordinates to evaluate:

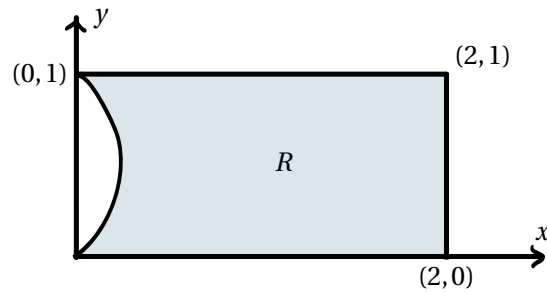
$$\iint_R x - 2y \, dA$$



- (a) Find a linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  which takes the unit square  $S$  to  $R$ . Write your answer both as a matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  and as  $T(u, v) = (au + bv, cu + dv)$ , and check your answer with the instructor.
- (b) Compute  $\iint_R x - 2y \, dA$  by relating it to an integral over  $S$  and evaluating that. Check your answer with the instructor.
2. Another simple type of transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a translation, which has the general form  $T(u, v) = (u + a, v + b)$  for a fixed  $a$  and  $b$ .
- (a) If  $T$  is a translation, what is its Jacobian matrix? How does it distort area?
- (b) Consider the region  $S = \{u^2 + v^2 \leq 1\}$  in  $\mathbb{R}^2$  with coordinates  $(u, v)$ , and the region  $R = \{(x - 2)^2 + (y - 1)^2 \leq 1\}$  in  $\mathbb{R}^2$  with coordinates  $(x, y)$ . Make separate sketches of  $S$  and  $R$ .
- (c) Find a translation  $T$  where  $T(S) = R$ .
- (d) Use  $T$  to reduce
- $$\iint_R x \, dA$$
- to an integral over  $S$ , and then evaluate that new integral using polar coordinates.
- (e) Check your answer in (d) with the instructor.

Problems 3 and 4 on the back.

3. Consider the region  $R$  shown below. Here the curved left side is given by  $x = y - y^2$ . In this problem, you will find a transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  which takes the unit square  $S = [0, 1] \times [0, 1]$  to  $R$ .



- (a) As a warm up, find a transformation that takes  $S$  to the rectangle  $[0, 2] \times [0, 1]$  which contains  $R$ .
- (b) Returning to the problem of finding  $T$  taking  $S$  to  $R$ , come up with formulas for  $T(u, 0)$ ,  $T(u, 1)$ ,  $T(0, v)$ , and  $T(1, v)$ . Hint: For three of these, use your answer in part (a).
- (c) Now extend your answer in (b) to the needed transformation  $T$ . Hint: Try “filling in” between  $T(0, v)$  and  $T(1, v)$  with a straight line.
- (d) Compute the area of  $R$  in two ways, once using  $T$  to change coordinates and once directly.
4. If you get this far, evaluate the integrals in Problems 1 and 2 directly, without doing a change of coordinates. It’s a fun-filled task . . .