

Tuesday, November 12 ** *Parametrizations and Integrals*

1. Consider the ellipsoid with implicit equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

- (a) Parameterize this ellipsoid.
(b) Set up, but do not evaluate, a double integral that computes its surface area.

2. Let

$$\mathbf{r}(u, v) = \langle (2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u \rangle,$$

where $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$.

- (a) Sketch the surface parameterized by this function.
(b) Compute its surface area.

3. Consider the surface integral

$$\iint_{\Sigma} z \, dS$$

where Σ is the surface with sides S_1 given by the cylinder $x^2 + y^2 = 1$, S_2 given by the unit disk in the xy -plane, and S_3 given by the plane $z = x + 1$. Evaluate this integral as follows:

- (a) Parameterize S_1 using (θ, z) coordinates.
(b) Evaluate the integral over the surface S_2 without parameterizing.
(c) Parameterize S_3 in (Des)cartesian coordinates and evaluate the resulting integral using polar coordinates.

4. Let C be the circle in the plane with equation $x^2 + y^2 - 2x = 0$.

- (a) Parameterize C as follows. For each choice of a slope t , consider the line L_t whose equation is $y = tx$. Then the intersection $L_t \cap C$ of L_t and C contains two points, one of which is $(0, 0)$. Find the other point of intersection, and call its x - and y -coordinates $x(t)$ and $y(t)$. Compute a formula for $\mathbf{r}(t) = \langle x(t), y(t) \rangle$. Check your answer with your TA.

- (b) Suppose that $t = \frac{p}{q}$ is a rational number. Show that $x(p/q)$ and $y(p/q)$ are also rational numbers. Explain how, by clearing denominators in $x(p/q) - 1$ and $y(p/q)$, you can find a triple of integers U, V , and W for which $U^2 + V^2 = W^2$.

- (c) Compute $\int_C \frac{1}{2} \langle -y, x \rangle \cdot d\mathbf{r}$ using your parameterization above.