**Thursday, December 5**   ∗∗   *Stokes' Theorem*

1. Let $S$ be the portion of the cylinder of radius 2 about the $x$-axis where $-1 \leq x \leq 1$.

    (a) Draw a picture of $S$ and compute its area without doing any integrals. Hint: How could you make this cylinder out of paper?

    (b) Find a parameterization $\mathbf{r}(u, v)$ of $S$.

    (c) Does the normal vector field associated to your parameterization point into or out of $S$? First, try to determine this without doing any calculations, and then check your answer by evaluating $\mathbf{r}_u \times \mathbf{r}_v$.

    (d) If necessary, change your parameterization so that the normal vector field points *inwards*.

    (e) Now consider the vector field $\mathbf{F} = \langle -z, \ xz, \ -xy \rangle$. Compute curl $\mathbf{F}$.

    (f) Check that curl $\mathbf{F}$ is the sum of $\mathbf{G} = \langle -2x, \ -1, \ 0 \rangle$ and $\mathbf{H} = \langle 0, \ y, \ z \rangle$.

    (g) Use geometric arguments to determine whether the flux of $\mathbf{G}$ is positive, zero, or negative. Remember that we have oriented $S$ so that the normals point inwards. Do the same for $\mathbf{H}$ and curl $\mathbf{F}$.

    (h) Using your parametrization, directly compute the flux of curl $\mathbf{F}$.

    (i) Check your answer in (h) using Stokes' Theorem. Note here that $\partial S$ has two boundary components, and make sure that your orient them correctly.

    (j) Check your answer in (h) a second time by using what you learned in (g) to compute the flux of $\mathbf{G}$ and $\mathbf{H}$.

2. Consider the surface $S$ shown below, which is oriented using the outward pointing normal.
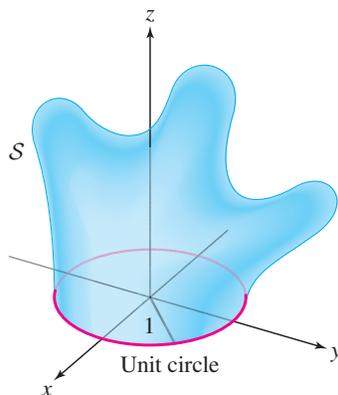


    (a) Suppose $\mathbf{F}$ is a vector field on $\mathbb{R}^3$ which is equal to curl $\mathbf{G}$ for some unknown vector field $\mathbf{G}$. Suppose the line integral of $\mathbf{G}$ around the unit circle (oriented counter-clockwise) in the $xy$-plane is 25. Determine the flux of $\mathbf{F}$ through $S$.

    (b) Suppose $\mathbf{H}$ is a vector field on $\mathbb{R}^3$ which is equal to curl $\mathbf{B}$ for some unknown vector field $\mathbf{B}$. If $\mathbf{H}(x, y, 0) = \mathbf{k}$, find the flux of $\mathbf{H}$ through the surface $S$.

    Check your answers with the instructor.