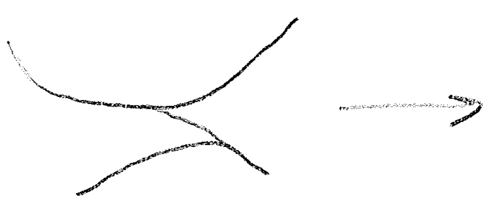
Lecture 22: Branched surfaces.

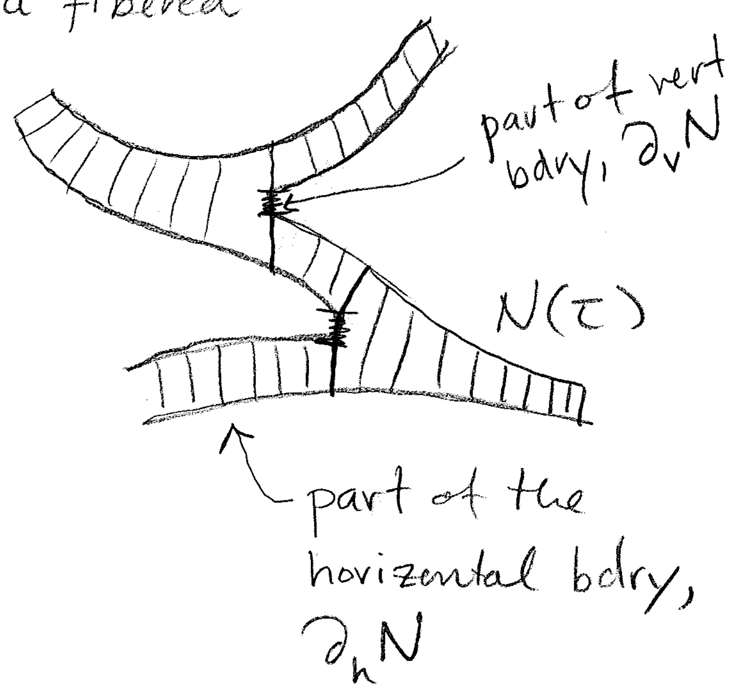
Last time: A lamination Λ of M^3 is a closed subset foliated by surfaces (charts where Λ is $\mathbb{R}^2 \times K$ with K cpt or $\mathbb{R}$). Λ is essential when a) no leaf is S^2 or a torus bounding a solid torus and b) each comp region R is irred, ∂R is incomp and end incomp.

Thm: If clsd orient M^3 has an ess. lam Λ, then M is irred, the leaves of Λ are incomp, and tight trans are $\neq 1$ in $\pi_1 M$.

Standing assumptions: M^3 clsd, orient, irred

A tram track τ in S has a fibered nbhd:





Fibers are intervals — collapse these to recover τ.

A curve C or lamn Λ is carried by τ when it is isotopic into $N(\tau)$ where it is trans. to the I-fibers. It is <u>fully carried</u> when it meets <u>every</u> I-fiber.

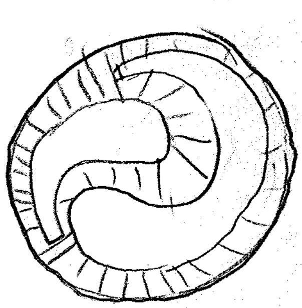
<u>Prop</u>: Suppose no comp. component of $N(\tau)$ is a disc D which meets $\partial_v N(\tau)$ in ≤ 1 comps. Then every multi curve C carried by τ is essential.

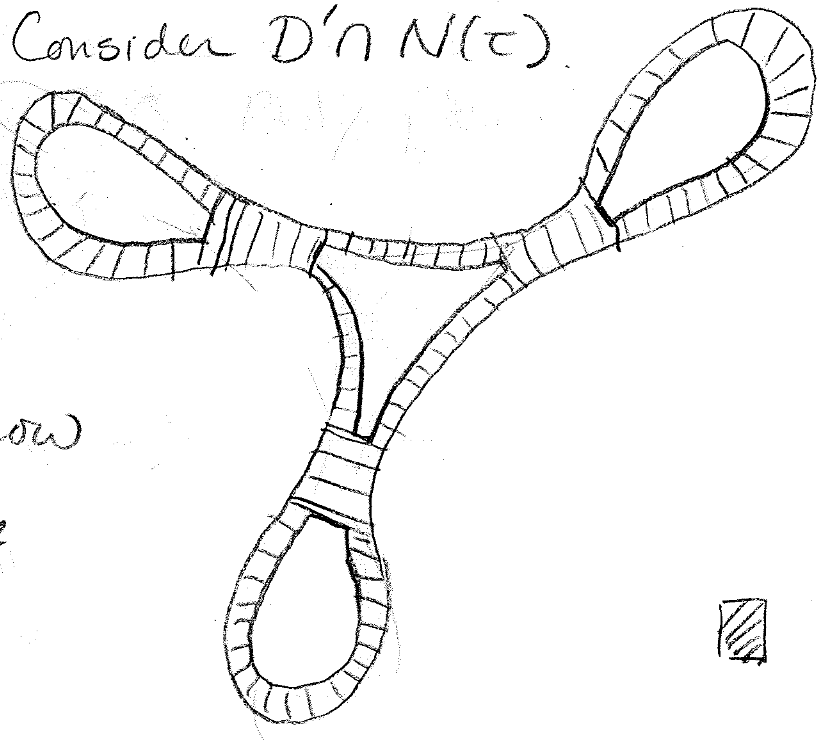
OK:

<u>NOT OK</u>:

<u>Pf</u>: If not, let D' be a disc with $D' \cap C$ a component C_0 of C. Consider $D' \cap N(\tau)$.

Some poss.

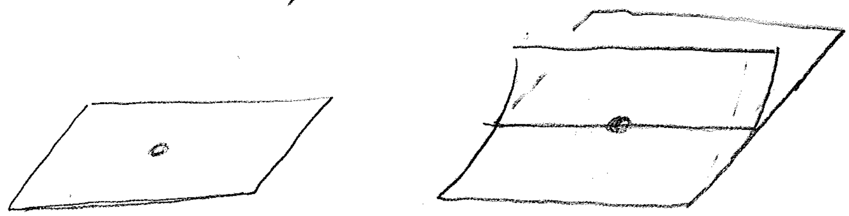


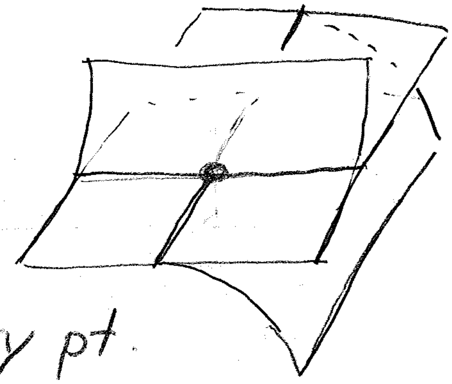


Use Euler char to show that must always have a bad region.

∎

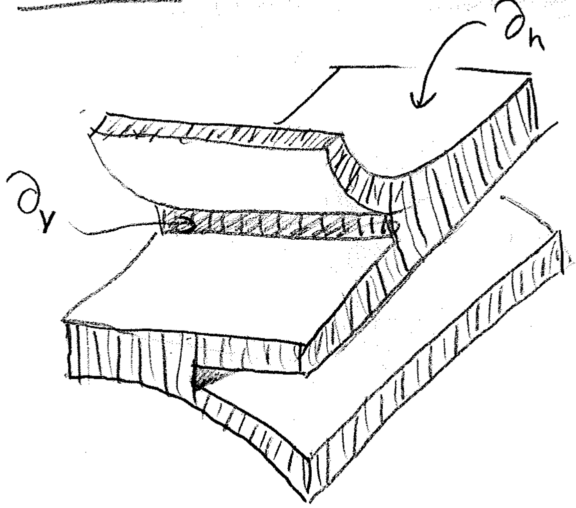
A <u>branched surface</u> $B \subseteq M^3$ is a closed subset locally like one of:

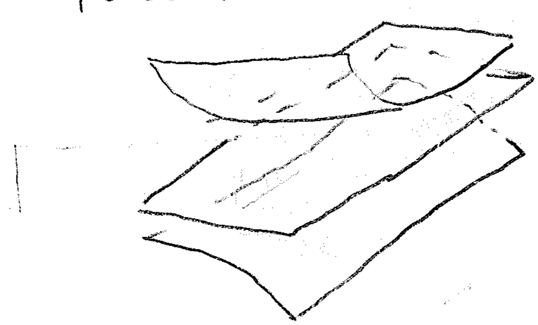




Has a well-def tangent plane at every pt.

Singular locus are pts in B w/o a nbhd $\cong \mathbb{R}^2$ — it is a 4-valent graph. $B \setminus (\text{sing loc})$ is a collection of surfaces with boundary, called <u>sectors</u>. Has a fibered nbhd $N(B)$ which loc. carries three surfaces:

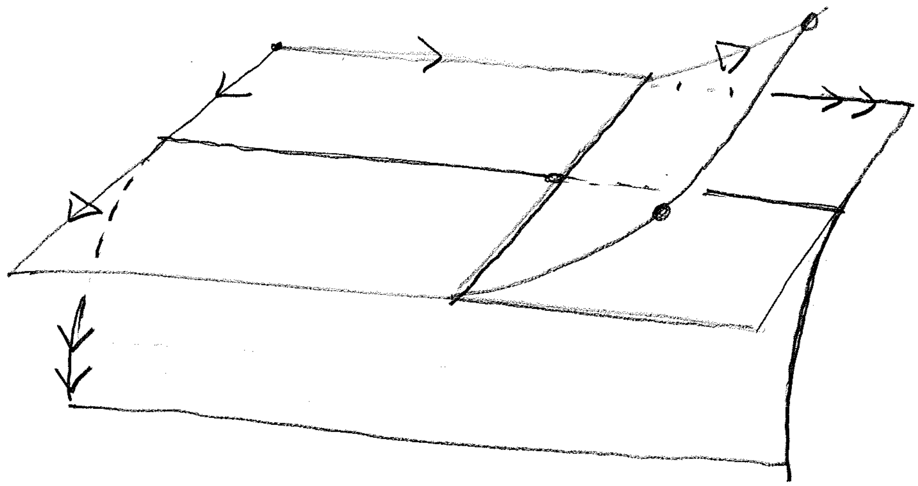




Given weights on the sectors sat the switch conditions, get a surface carried by $N(B)$.

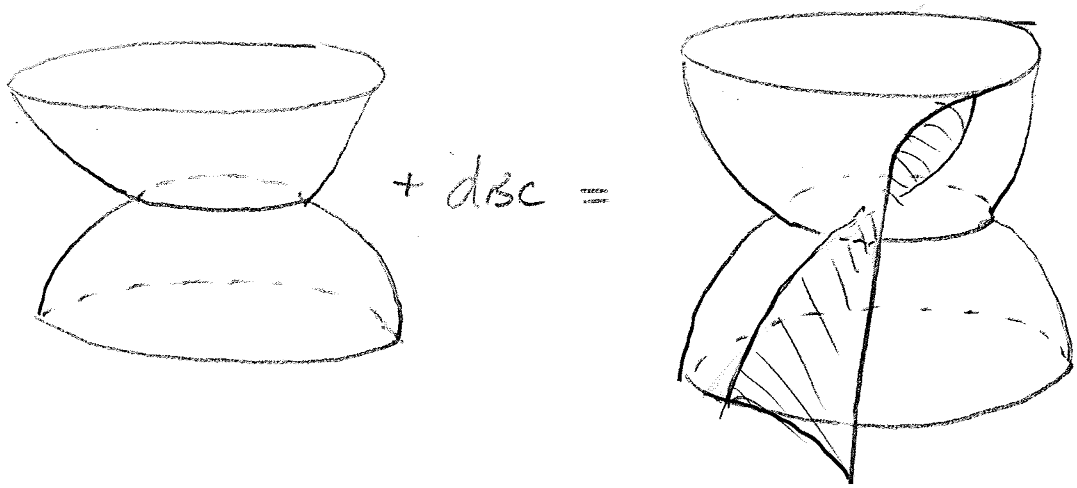
Unlike train tracks, not every branched surface fully carries a lamination.

Twisted disk of contact:

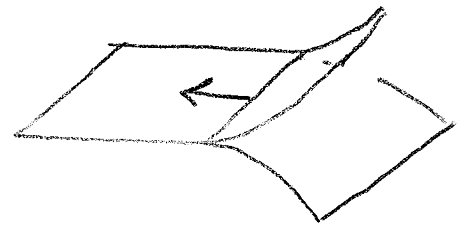


Can't be any leaves cor. to the horizontal plane because of holonomy issues.

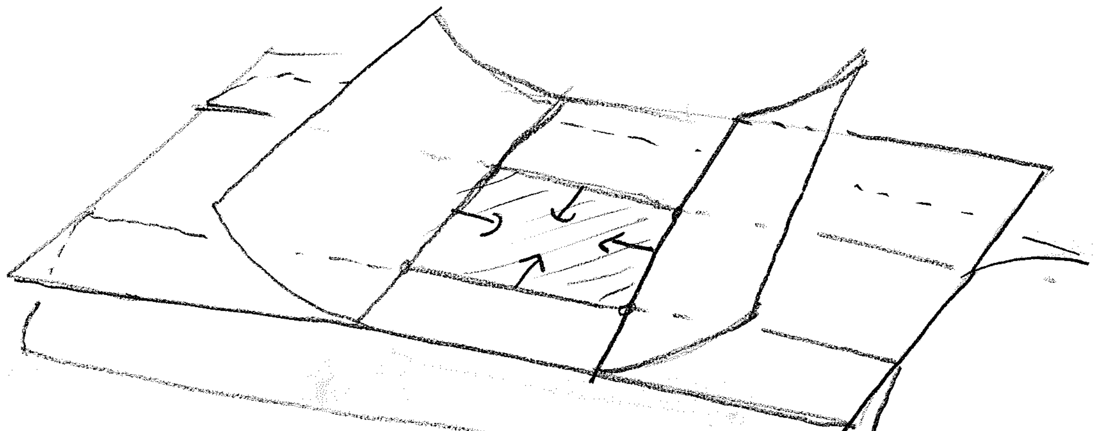
Secretly:



Each comp of the sing. locus of B has a branch direction (maw vector field)



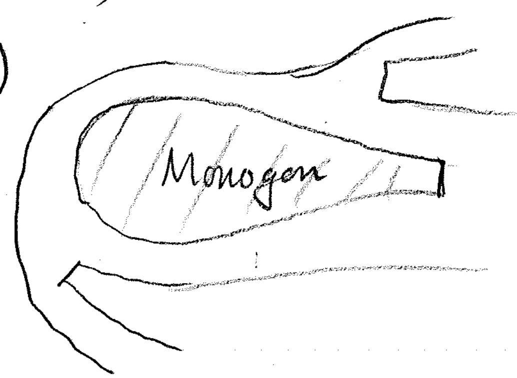
A sink disc is a disc sector where this points inwards everywhere.



A branched surface is laminar when

1) $\partial_h N(B)$ is incomp in $M \setminus \mathring{N}(B)$, no component of $\partial_h N(B)$ is S^2, and $M \setminus B$ is irred.

2) No monogon in $M \setminus \mathring{N}(B)$



3) B does not carry a torus bounding a solid torus

4) No sink discs (after collasping bubbles)

[Tao Li] Every laminar branched surfaces fully carries an ess. lamination.