Recall: $H^2 = \{ |z| < 1 \mid z \in \mathbb{C} \}$

$S^\infty_1$ = equiv classes of good rays

[Candel] If $\mathcal{F}$ is a taut fol of an irreducible atoroidal orient $M^3$, then $\exists$ metric on $M$ s.t. every leaf has constant curvature $-1$.

[Q: What goes wrong for toroidal $M$?]

Cor: $\mathcal{F}$ of $\tilde{M}$ is a fol by hyp planes.

Goal: Unify the $S^1_\infty(\tilde{\mathcal{F}})$ into one circle with a $\pi_1 M$ act.

Note: Intrinsic geom of $\tilde{\mathcal{F}}$ is $H^2$; extrinsic geom can be crazy. For $M_f$, each $\tilde{\mathcal{F}} \subseteq H^3$ limits to all of $S^\infty_\infty$. Get $\pi_1 F$ equiv. space-filling curve $S^1_\infty(\tilde{\mathcal{F}}) \to S^2_\infty(\tilde{M})$ [Cannon-Thurston].
Two issues:

1) Non-Hausdorff leaf space.

Some leaves are "incomparable": no trans. meets both.

2) Even comparable leaves may not be uniformly close. Suppose $\gamma$ is a good ray in $\tilde{\mathcal{L}}$ starting at $p$.

Can we find a transv. $\tau$ at $p$ so that the hol of $\tau$ is defined along the whole of $\gamma$?

Marker: $I \times \mathbb{R}^+ \to \tilde{M}$ where

a) each $t \times \mathbb{R}^+$ is a good ray in a leaf $\tilde{L}$.

b) each $I \times s$ is a transv to $\mathcal{F}$ of len $\leq \varepsilon_0$. 
Leaf Pocket Thm: For every \( \overline{L} \) of \( \mathcal{F} \), the end pts of markers are dense in \( S^1_{\infty}(\overline{L}) \).

Thurston proved this using harmonic measures on the leaves to study the behavior of random walks on a fixed leaf. Cartoon of idea: A transv. where

\[
\begin{array}{c}
\varepsilon \\
\hline
\varepsilon \\
\hline
\varepsilon
\end{array}
\]

expands by a factor of 3 \( \Rightarrow 3 \) transv.

where the holonomy contracts by 1/3. Thus contract.

holonomy is generic. Will instead follow [CD 2003]

Idea: 1) Some leaf \( L \) of \( \mathcal{F} \) has \( \pi_1 L \neq 1 \). Otherwise, no holonomy \( \Rightarrow \) leaves of \( \mathcal{F} \) have poly. area growth.

[Plante]

This is a contradiction as each leaf is \( \# L^2 \)
in this scenario.
3) Argue about minimal sets.

(e.g. suppose every leaf is dense.)

A universal circle \( S'_{univ} \) for \( f \) is

a) An action of \( \pi_1 M \) on \( S'_{univ} \) \( (\pi_1 M \to \text{Homeo}(S'_{univ})) \)

b) For every leaf \( \tilde{\gamma} \) a monotone map

\[
\phi_{\tilde{\gamma}} : S'_{univ} \to S'_\infty(\tilde{\gamma})
\]

where \( \forall \tilde{\gamma} \) and \( \alpha \in \pi_1 M \) one has

\[
\begin{array}{ccc}
S'_{univ} & \xrightarrow{\text{Puniv}(\alpha)} & S'_{univ} \\
\phi_{\tilde{\gamma}} \downarrow & & \downarrow \phi_{\alpha(\tilde{\gamma})} \\
S'_\infty(\tilde{\gamma}) & \xrightarrow{\alpha} & S'_\infty(\alpha(\tilde{\gamma}))
\end{array}
\]
Thm: \( F \) taut fol of irreducible \( \text{closd } M^3 \). Then \( F \) has a univ. circle. Moreover, the action of \( \pi_i M \) on \( S'_{\text{univ}} \) is faithful \( \Rightarrow \pi_i M \) is a subgp of \( \text{Homeo}(S') \).

Idea: Use markers to take an inverse limit of the actions on the individual \( S'_{\infty}(\mathbb{Z}) \).

For more, see Calegari's book or [Calegari-D 2003].