Suppose $S$ is a hyperbolic surface with any geodesic ideal tri $I$ and fixed cusp nbhds $C_i$.

Have a $\pi_1 S$ equiv map $\bar{I} \to \mathbb{R}^2$ senting each ideal tri to a linear one w/ roots in $V$, where $V \subseteq L^+$ cor. to $C_i$.

Gives a "bent plane" $X$ inside $L^+$ that projects out to the orig. geod. ideal tri.

The orig. tri. is canonical $\iff X = \partial P$, $P$ convex hull of $V$. $\iff X$ bounds a convex region inside $L^+$ $\iff$ each pair of tri meeting along an edge "fold up" not down as in $\bigcirc$
Purely local and only need check for one edge of $X$ in each $\Pi_i S$-orbit, i.e., once for each edge of $J$ down stairs. [Aside about Penner coor, horoball decorations...]

If pair folds down, do a move along this edge to create $J_1$.

Get a seq $J_1, J_2, \ldots$ with cor $X_1, X_2, \ldots$ inside $L^*$ that "move down."

Prop: This must terminate.

An edge class is a segment joining two elts in $V$, together with its $\Pi_i S$ orbit.

Claim: Only finitely many edge classes below the orig. $X$.

Only finitely many vertex classes, so suffices to bound the number of edge classes ending at a fixed $v_0 \in V$. 
Consider $f: X \to \mathbb{R}$ by $f(x) = \langle x, x \rangle$ which is cont. and $\Pi_1 S$ equiv. It is bounded since

\[ X = \text{compact, } \text{of } S \text{ adding one pt at cusp} \]

Say $f(x) \leq [-r^2, 0]$

Lemma: $u, v \in L^+$. Then

a) $\langle u - v, u - v \rangle = -2 \langle u, v \rangle \geq 0$

b) $\min \langle x, x \rangle$ for $x = (1-t)u + tv$ $t \in [0,1]$ is at the mid pt $\langle \frac{u+v}{2}, \frac{u+v}{2} \rangle = \langle \frac{u+v}{2} \rangle$

c) The dist between the horocircles $H_u$ and $H_v$ is $\log \left( \frac{\langle u, v \rangle}{-2} \right)$.

Pf: a) is because $u - v$ is space-like.

b) is since $\langle x, x \rangle = 2(1-t)t \langle u, v \rangle$ and $\langle u, v \rangle \leq 0$.

c) Transform to reduce to $u = (t, 0, t)$ and $v = (-t, 0, t)$ and see $\text{dist}(H_u, H_v) = 2 \log t = \log t^2 = \log \left( \frac{\langle u, v \rangle}{-2} \right)$.

Pf of claim: An edge $e$ from $V_0$ to $V_1$ lying below $X$ must sat $\langle x, x \rangle \geq -r^2$ along the seg $(V_0, V_1)$

$\Rightarrow \langle V_0, V_1 \rangle \geq -r^2 \Rightarrow \text{dist}(H_{V_0}, H_{V_1}) \leq \log \frac{r^2}{2}$

Now there are only finitely many orbits of
such horoballs \( H_{v_i} \):
and hence finitely many such edge classes.

This proves the claim and hence the prop.  

Remarks: 1) To get final cellulation have to erase any edges \( \square \rightarrow \square \) where there is a fold.

2) For \( n=2 \), the above proves the lemma about finiteness of faces of the canonical cellulation that I skipped last time.

Case of 3-mflds is very similar, with flip replaced by \( 2 \rightarrow 3 \) and \( 3 \rightarrow 2 \) moves.

Still a local test on faces and valence 3 edges, depending on whether things are
concave/convex in $\mathbb{R}^{3,1}$ inside the light cone.
For details, see Week's paper.

New issue: creation of neg. orient tets.

[Diagrams of 2D and 3D geometrical transformations]

Now the "move down" algorithm can get stuck.