

Math 418: //DRAFT ONLY// HW 4 due Wednesday, February 23, 2022.

Webpage: <http://dunfield.info/418>

Office hours: Monday and Tuesday from 1:30-2:30pm; other times possible by appointment.

1. Let K/F be an algebraic extension. Suppose R is a *subring* contained in K which contains F . Prove that R is actually a *subfield* of K .
2. Prove that $\alpha = \cos(2\pi/5)$ is a constructable number. Use this to show that the regular 5-gon is constructable by straightedge and compass.
3. Find the splitting field K of $x^4 - 2$ over \mathbb{Q} . What is $[K : \mathbb{Q}]$?
4. Find the splitting field K of $x^4 + x^2 + 1$ over \mathbb{Q} . What is $[K : \mathbb{Q}]$?
5. Suppose K/F is the splitting field for a polynomial $f(x) \in F[x]$. Let $g(x) \in F[x]$ be irreducible. Show that if g has a root in K then it splits completely in $K[x]$.