1. Let $K_1$ and $K_2$ be subfields of some ambient field $L$ which both contain a subfield $F$. Suppose that each $K_i$ is the splitting field of a polynomial $f_i \in F[x]$. Prove that $K_1 \cap K_2$ is also the splitting field of some polynomial in $F[x]$.
   
   Hint: Problem 5 from HW #4.

2. Find all irreducible polynomials of degree 1, 2, and 4 over $\mathbb{F}_2$. Check that their product is $x^{16} + x$.

3. Suppose $K$ is an extension of a perfect field $F$. Suppose that $f(x) \in F[x]$ has no repeated irreducible factors in $F[x]$. Prove that it also has no repeated irreducible factors in $K[x]$.

4. A field element $\zeta$ is called a root of unity if $\zeta^n = 1$ for some $n > 0$. Using the results of Section 13.6, prove that if $K$ is a finite extension of $\mathbb{Q}$ then it contains only finitely many roots of unity.