Math 418: //DRAFT ONLY// HW 5 due Wednesday, March 2, 2022.

Webpage: `http://dunfield.info/418`

Office hours: Monday and Tuesday from 1:30–2:30pm; other times possible by appointment.

1. Let K_1 and K_2 be subfields of some ambient field L which both contain a subfield F. Suppose that each K_i is the splitting field of a polynomial $f_i \in F[x]$. Prove that $K_1 \cap K_2$ is also the splitting field of some polynomial in $F[x]$.

   Hint: Problem 5 from HW #4.

2. Find all irreducible polynomials of degree 1, 2, and 4 over $\mathbb{F}_2$. Check that their product is $x^{16} + x$.

3. Suppose K is an extension of a perfect field F. Suppose that $f(x) \in F[x]$ has no repeated irreducible factors in $F[x]$. Prove that it also has no repeated irreducible factors in $K[x]$.

4. A field element ζ is called a root of unity if $\zeta^n = 1$ for some $n > 0$. Using the results of Section 13.6, prove that if K is a finite extension of $\mathbb{Q}$ then it contains only finitely many roots of unity.