

**Math 418: //DRAFT ONLY// HW 10 due Wednesday, May 4, 2022.**

**Webpage:** <http://dunfield.info/418>

**Final Exam:** Our final will be Wednesday, May 11 from 1:30–4:30pm in our usual classroom. The exam will be comprehensive, but significant extra weight will be put on material covered after the *in-class* midterm, i.e. Galois theory and algebraic geometry. Expect roughly double the number of questions as the midterm, but you'll have 3 full hours, rather than 50 minutes.

**Cheat sheet:** You are allowed **two** sheets of standard size paper, on which you can write, print, photocopy, etc. anything that you think will be helpful on the exam.

**Office hours bonus week:**

- TBA.

**Actual assignment:**

1. Let  $V \subset k^n$  be an affine algebraic variety over an algebraically closed field  $k$ . Prove that  $f \in k(V)$  lies in  $k[V]$  if and only if  $\text{dom}(f) = V$ .
2. Consider the affine plane curve  $X = \mathbf{V}(x^3y + y^3 + x)$  in  $\mathbb{C}^2$ , which is an affine part of the curve from Problem 4 from HW 9, so in particular it is smooth and irreducible. Consider the polynomial function  $f = x + y \in \mathbb{C}[X]$ . As discussed in class, this gives a field extension  $\mathbb{C}(X)/\mathbb{C}(t)$ . Compute the degree of this extension, and identify it with an abstract extension of the form  $\mathbb{C}(t)[u]/(p(u))$ .
3. Do any problem from Dummit and Foote, or Cox et. al., or Reid, or really any book at all, that you think will help you prepare for the final exam.
4. Repeat problem 3.
5. Repeat problem 3.